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Lead monoxide: a two-dimensional ferromagnetic semiconductor induced by hole-doping

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We employ first-principles calculations to demonstrate ferromagnetic ground states for single- and multi-layer lead monoxide (PbO) under hole-doping, originating from a van Hove singularity at the valence band edge. Both the sample thickness and applied strain are found to have huge effects on the electronic and magnetic properties. Multi-layer PbO is an indirect band gap semiconductor, while a direct band gap is realized in the single-layer limit. In hole-doped single-layer PbO, biaxial tensile strain can enhance the stability of the ferromagnetic state.

Introduction

Nowadays extensive research efforts focus on the development of novel two-dimensional (2D) materials, including semi-conducting metal monochalcogenides MX (M = Ge, Sn; X = S, Se, Te)^{1–8} and transition metal dichalcogenides TX₂ (T = Mo, W; X = S, Se, Te),^{9–16} with properties superior to the corresponding bulk compounds. Theoretical studies have shown that single-layer MX and TX₂ exhibit wide band gaps due to quantum confinement, giant spin splitting due to inversion symmetry breaking, and strong spin-orbit coupling.^{17–20} An ultimate goal of 2D semiconductor research is control of the spin degree of freedom to design spintronics devices that utilize both the charge and spin of carriers.^{21–23} Magnetism in 2D materials can be achieved by doping of magnetic atoms,^{24,25} defects,^{26–28} and strain engineering.^{29,30} Recently, single-layer GaSe and GaS^{31,32} have been reported to develop ferromagnetism under hole doping.

Metal oxides, the largest class of compounds on earth, are widely known for exotic magnetism and superconductivity, for example,^{33–36} but not for 2D semiconductors. Therefore, it is of great interest that Singh and Hennig³⁷ have demonstrated that single-layer lead monoxide (PbO) has a stable tetragonal litharge structure with a direct band gap. Because a systematic study of PbO is missing, we investigate in the present work the influence of the thickness as well as the effects of strain and hole doping on the electronic structure by means of *ab initio* calculations.

Methods

Ab initio calculations are performed using density functional theory in the generalized gradient approximation (GGA) with Perdew–Burke–Ernzerhof parametrization for the exchange–correlation functional with van der Waals correction,^{38,39} as implemented in the Quantum-ESPRESSO package.⁴⁰ We employ ultrasoft pseudopotentials and a 40 Ry plane wave energy cutoff. The in-plane lattice constant is 4.01 Å, and a 20 Å thick vacuum slab is used to model single- and multi-layer PbO. The atomic coordinates are fully relaxed until all atomic forces have decreased below 0.01 eV Å^{−1}. For bulk and layered PbO, respectively, the Brillouin zone is sampled on a 6 × 6 × 4 and 6 × 6 × 1 *k*-point grid.

Results and discussion

Single-layer PbO has a tetragonal structure, as shown in Fig. 1(a and b), with Pb connected to four O atoms in a pyramidal form. Bulk and multi-layer PbO are constructed by stacking these layers, see Fig. 1(c). The 2D Brillouin zone is depicted in Fig. 1(d). According to Fig. 2, the band gap increases when the number of layers decreases. Bulk and multi-layer PbO exhibit

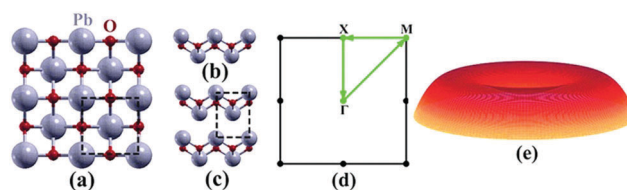


Fig. 1 (a) Top view of PbO. (b and c) Side views of single-layer and bulk PbO. (d) 2D Brillouin zone with high symmetry points. (e) Schematic representation of a Mexican-hat dispersion.

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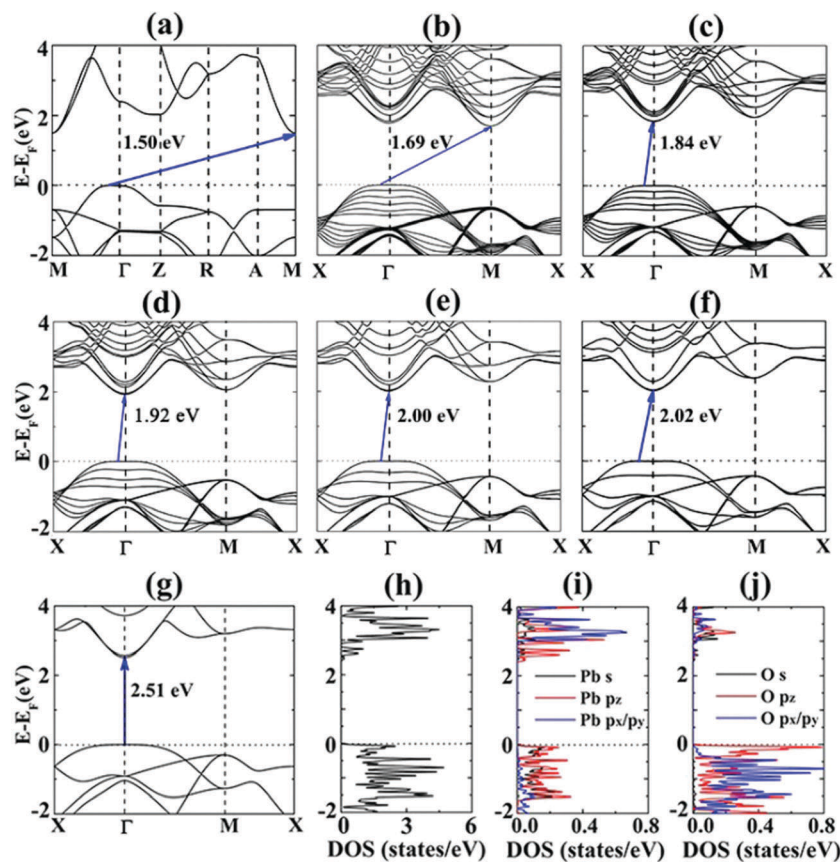


Fig. 2 Band structure of (a) bulk, (b) sextuple-, (c) quintuple-, (d) quadruple-, (e) triple-, (f) double-, and (g) single-layer PbO. The energy difference between the VBM and CBM is given. (h–j) DOS of single-layer PbO.

indirect band gaps, while transition to a direct band gap occurs in the single-layer limit. In the case of sextuple-layer PbO, Fig. 2(b) shows that the conduction band minimum (CBM) at the *M* point is only 0.09 eV below that at the Γ point. Reducing the number of layers shifts it to the Γ point. A Mexican-hat dispersion with a ring-like Fermi surface, see the schematic representation in Fig. 1(e), results in a van Hove singularity at the band edge.^{25,31} This instability can induce ferromagnetism, superconductivity, and other phase transitions. Mexican-hat dispersions have been reported for many 2D materials,⁴¹ including single-layer GaSe and GaS.^{1,6} Fig. 2 demonstrates that the valence band maximum (VBM) stays near the Γ point for multi-layer PbO, reflecting a Mexican-hat dispersion. The energy difference between the VBM and highest occupied band at the Γ point is small, less than 8 meV, see Fig. 2(b–g). In the single-layer limit the VBM shifts to the Γ point, resulting in a direct band gap. Although the Mexican-hat dispersion disappears for single-layer PbO, the dispersion is rather flat at the VBM and the density of states (DOS) thus is high, as shown in Fig. 2(h). The DOS projected on the Pb and O atoms, see Fig. 2(i and j), indicates that the conduction band edge is largely due to the Pb p_z orbitals and the valence band edge due to the O p_z and Pb s, p_z orbitals.

Interestingly, we find that the band gap of single-layer PbO can be transformed from direct to indirect by biaxial tensile strain. Fig. 3(a–d) shows band structures for 3–6% strain.

A transition occurs close to 3%, because the highest occupied band develops a dip at the Γ point (with the CBM remaining at the Γ point). This gives rise to a Mexican-hat dispersion similar to multi-layer PbO. The depth of the dip at the Γ point grows rapidly from 1 meV at 4% strain to 20 meV at 6% strain. Fig. 3(e–g) addresses the DOS under 6% strain to demonstrate that the orbital characters of the valence and conduction band edges have not changed. By comparing Fig. 2(h) and 3(e) we find that the DOS at the valence band edge (and thus the electronic instability) is enhanced by strain.

While unstrained single-layer PbO is nonmagnetic, the van Hove singularity at the valence band edge may induce ferromagnetism when the Fermi level is tuned by hole doping, which is confirmed by our calculations. The spin polarized total DOS of unstrained single-layer PbO with hole density $1.6 \times 10^{14} \text{ cm}^{-2}$ is depicted in Fig. 4(a) and that of single-layer PbO under 6% biaxial tensile strain with hole density $2.0 \times 10^{14} \text{ cm}^{-2}$ in Fig. 4(b). We observe in both cases a large exchange splitting. Hole doping alone maintains the orbital character of the valence band edge (O p_z and Pb s, p_z orbitals), whereas combination with strain enhances the weight of the O p_z and Pb s contributions, see Fig. 4(b–f), and the spatial localization of the states. It is worth noting that biaxial tensile strain strongly enhanced the exchange splitting (from 0.22 eV to 0.32 eV under 6% strain), see Fig. 4(a and b), and thus the stability of the ferromagnetic state.

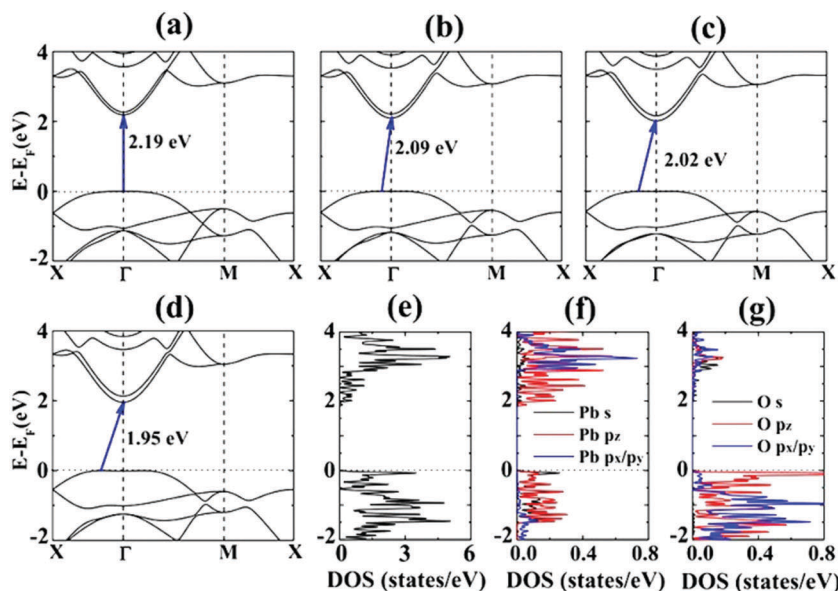


Fig. 3 Band structure of single-layer PbO under (a) 3%, (b) 4%, (c) 5%, and (d) 6% biaxial tensile strain. (e–g) DOS under 6% biaxial tensile strain.

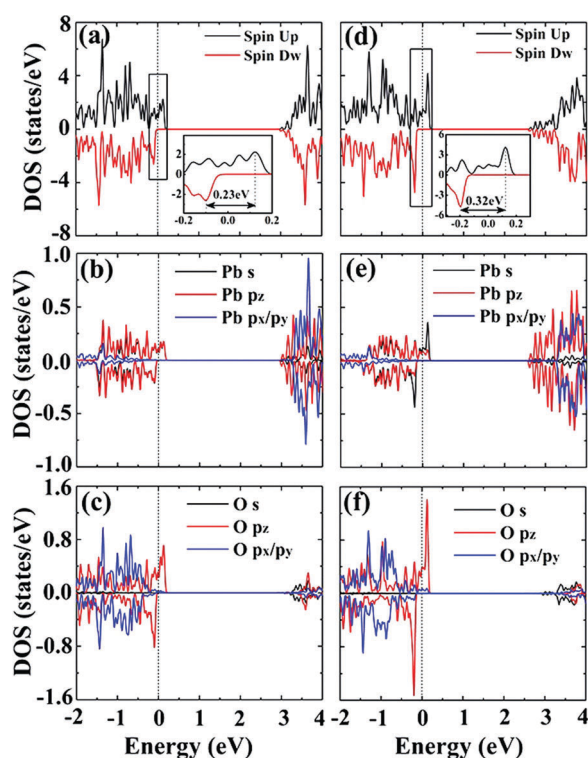


Fig. 4 Spin-resolved DOS of (a–c) unstrained single-layer PbO with hole density $1.6 \times 10^{14} \text{ cm}^{-2}$ and (d–f) single-layer PbO under 6% biaxial tensile strain with hole density $2.0 \times 10^{14} \text{ cm}^{-2}$.

Calculating the total energy for nonmagnetic (E_{NM}), ferromagnetic (E_{FM}), and antiferromagnetic (E_{AFM}) phases, we check the stability of magnetization by determining the polar magnetic energy ΔE_{mag} as the energy difference $E_{\text{NM}} - E_{\text{FM}}$ for single-layer, and $E_{\text{AFM}} - E_{\text{FM}}$ for double- or triple-layer PbO.³⁵ A positive ΔE_{mag} reflects a preference for ferromagnetic ordering.

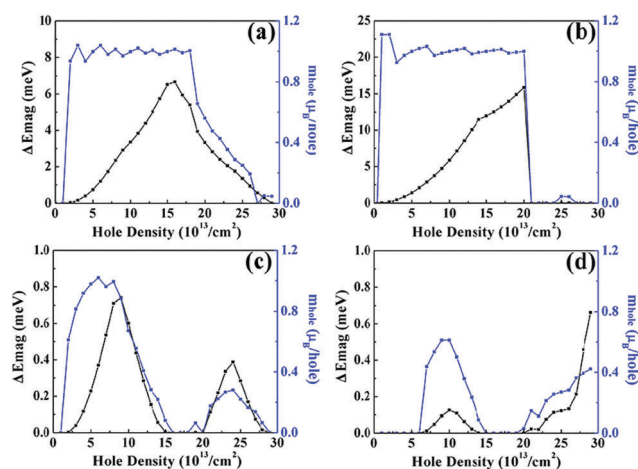


Fig. 5 Dependence of the polarization energy (black) and magnetic moment (blue) on the hole density: (a) unstrained single-layer PbO, (b) single-layer PbO under 6% biaxial tensile strain, (c) unstrained double-layer PbO, and (d) unstrained triple-layer PbO.

Fig. 5(a) shows that single-layer PbO turns ferromagnetic when the hole density reaches $0.2 \times 10^{14} \text{ cm}^{-2}$, with a magnetic moment of $m = 0.93 \mu_{\text{B}}$ per hole. We find little changes up to a hole density of $1.9 \times 10^{14} \text{ cm}^{-2}$ and thereafter a slow reduction of m to zero. At a hole density of $2.7 \times 10^{14} \text{ cm}^{-2}$ the nonmagnetic state is restored. In the plateau region of m , ΔE_{mag} shows a distinct maximum of 6.7 meV at a hole density of $1.6 \times 10^{14} \text{ cm}^{-2}$ (highest stability of the ferromagnetic state). Under 6% biaxial tensile strain single-layer PbO turns ferromagnetic at a lower hole density of $0.1 \times 10^{14} \text{ cm}^{-2}$, with $m = 1.1 \mu_{\text{B}}$ per hole, as shown in Fig. 5(b). We observe again a plateau region with $m \sim 1 \mu_{\text{B}}$ per hole, reaching up to a hole density of $2.0 \times 10^{14} \text{ cm}^{-2}$, i.e., added hole states carry equal magnetic moment. The maximum of ΔE_{mag} is reached at the

end of the plateau region and the value is much higher than for unstrained single-layer PbO, in agreement with the previous analysis of the DOS (enhancement of the electronic instability).

Fig. 5(c and d) addresses m and ΔE_{mag} for double- and triple-layer PbO, demonstrating the onset of ferromagnetism when the hole density reaches 0.2×10^{14} and $0.7 \times 10^{14} \text{ cm}^{-2}$, respectively. However, the ferromagnetism vanishes again at hole densities of 1.6×10^{14} and $1.5 \times 10^{14} \text{ cm}^{-2}$, respectively, *i.e.*, much earlier than for single-layer PbO. Double-layer PbO has a maximum of $m = 0.88 \mu_{\text{B}}$ per hole at a hole density of $0.9 \times 10^{14} \text{ cm}^{-2}$ and triple-layer PbO has a maximum of $m = 0.61 \mu_{\text{B}}$ per hole at a hole density of $1.0 \times 10^{14} \text{ cm}^{-2}$. ΔE_{mag} behaves similar to m in both cases, reaching 0.74 meV at a hole density of $0.9 \times 10^{14} \text{ cm}^{-2}$ for double-layer PbO and 0.13 meV at a hole density of $1.0 \times 10^{14} \text{ cm}^{-2}$ for triple-layer PbO. The fact that we find no plateau in m , in contrast to single-layer PbO, indicates that quantum confinement plays an important role in the magnetic properties of hole-doped PbO. We also notice that single-layer PbO achieves much higher values of ΔE_{mag} , suggesting that it is more promising as a stable ferromagnetic platform than hole-doped multi-layer PbO.

If ferromagnetism occurs then the exchange energy gain exceeds the kinetic energy loss, or in the language of the Stoner Criterion^{42,43} when $I\text{-DOS}(E_{\text{F}}) > 1$. Here E_{F} is the Fermi energy and I is the Stoner parameter that represents the strength of the exchange interaction.^{44,45} Since $\text{DOS}(E_{\text{F}}) \sim 0$ for single-layer PbO without hole doping, we have $I\text{-DOS}(E_{\text{F}}) < 1$ and thus a nonmagnetic state. When holes are introduced, the effective hole mass m_{h} determines $\text{DOS}(E_{\text{F}}) \propto m_{\text{h}}^{3/2}(E_{\text{VBM}} - E_{\text{F}})^{1/2}$. In the case of single-layer PbO the flat valence band edge results in large m_{h} . Therefore, $\text{DOS}(E_{\text{F}})$ increases with the hole density. In addition, the valence band edge of single-layer PbO is mainly due to O p orbitals, which host strong exchange interaction,³⁵ so that I is large for hole doping. Therefore, the product $I\text{-DOS}(E_{\text{F}})$ becomes large enough to turn single-layer PbO into a ferromagnet.

We estimate the Curie temperature (T_{c}) based on the mean-field approximation *via* $k_{\text{B}}T_{\text{c}} = 2/3\Delta E_{\text{mag}}$. The estimated T_{c} values for single-, double- and triple-layer PbO are 51 K, 6 K and 1 K, respectively. Because exchange splitting and the stability of the ferromagnetic state can be strongly enhanced by the biaxial tensile strain, the estimated T_{c} for strained single-layer PbO is 122 K, which is much higher than that of unstrained single- or multi-layer PbO.

In the present work, the electronic structures are calculated by using GGA, which always underestimates the band gap for insulators and semiconductors. We also calculate the band structure of single-layer PbO by using a hybrid functional approach and found that the band gap is 3.40 eV, which is 0.89 eV larger than the value from GGA. However, the band edge dispersion from the hybrid functional approach is very similar to that from GGA. Though Pb has large spin orbit interaction, the introduction of spin orbit interaction will reduce the calculated band gap a little bit, but has little effect on the magnetism of the PbO under hole doping and the easy axis of the magnetic moment keeps out of the plane.

Conclusion

In conclusion, we have employed first-principles calculations to investigate the influence of the sample thickness, applied strain, and hole doping on the electronic properties of PbO. For decreasing thickness both the VBM and CBM are found to shift to the Γ point, resulting in a direct band gap in the single-layer limit. We find for multi-layer PbO a Mexican-hat dispersion with indirect band gap, while single-layer PbO is characterized by a flat parabolic dispersion. Interestingly, the Mexican-hat dispersion reappears in single-layer PbO under biaxial tensile strain above 3%, inducing an indirect band gap. Both single- and multi-layer PbO develop half-metallicity under hole doping, where strain turns out to enhance the stability of the ferromagnetic state. *Via* mean-field approximation, we estimate a Curie temperature of $T_{\text{c}} = 122$ K for single-layer PbO under 6% biaxial tensile strain. We not only predict that ferromagnetism can be achieved in recently reported layered PbO, but also demonstrate that it originates from a van Hove singularity at the valence band edge. Our findings open a new avenue to realizing spintronics in single-layer oxides.

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